3-6 Videos Guide

3-6a

- The Ratio Test:
  - (i) If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L < 1$, then the series $\sum a_n$ is convergent
  - (ii) If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L > 1$ or is ∞, then the series $\sum a_n$ is divergent
  - (iii) If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 1$, the Ratio Test is inconclusive

Exercises:

Use the Ratio Test to determine whether the series is convergent or divergent.

3-6b

- $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$
- $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$

3-6c

- $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3-6d

- $\frac{2}{3} + \frac{2\cdot5}{3\cdot5} + \frac{2\cdot5\cdot8}{3\cdot5\cdot7} + \frac{2\cdot5\cdot8\cdot11}{3\cdot5\cdot7\cdot9} + \cdots$

3-6e

- The Root Test:
  - (i) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum a_n$ is convergent
  - (ii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or is ∞, then the series $\sum a_n$ is divergent
  - (iii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive

Exercises:

- Use the Root Test to determine whether the series is convergent or divergent.
  - $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$
  - $\sum_{n=1}^{\infty} \left(-\frac{2n}{n+1}\right)^{5n}$

3-6f

Exercises:

- Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

- $\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n}\right)^n$
- $\sum_{n=1}^{\infty} \frac{sin(n\pi/6)}{1+n\sqrt{n}}$